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$P(P^{-})$-nucleus Interactions at 200 GeV/c via Neural Network Technique

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Abstract— $P(P^{-})$ -nucleus interactions at 200 GeV/c have been studied. Two cases are considered; the charged pions multiplicity distribution and the negative ones. The neural net work (NN) technique has been adopted to study the same two cases, the trained NN shows a better fitting with experimental data than the PTFM calculations do. The NN simulation results are satisfactory and prove a vital and strong presence in modeling $P(P^{-})$ –nucleus interactions at 200 GeV/c. From paper to paper; we prove that the NN technique is better than the old conventional ones.

Index Terms— Artificial intelligence (AI), machine learning (ML), NN technique, multiplicity distribution, (h-A) interactions.

1 Introduction

Experimental data on hadron-nucleus (h-A) interactions at high energies are required for understanding high energy interactions. They provide a useful link between hadron-hadron (h-h) interactions and the complex phenomena of nucleus-nucleus (A-A) interactions. These types of interactions investigate space time picture and highlight on phenomenon which doesn't exist in (h-h) such as cascade, multi-collisions, gray particles, etc. There are various models for (h-A) interaction like collective tube model [1], diffractive excitation model[2], energy flux cascade model [3], quark model [4], interanuclear cascade model [5] multiple scattering model [6], hydrodynamical model [7], and many others .

From view point of parton two fireball model (PTFM), nucleons are treated as composite objects of loosely bound states of the spatially separated constituents (quarks) which in turn are composed of point-like particles (partons) [8]. This may allow one to consider the nucleons as consisting of a fixed number of partons. This nucleon structure has been used in different models [8-10] along with other assumptions to describe h-A interactions. PTFM, which is proposed by Hagedorn [11] and developed by Tantawy [12], has been used to explain the high energy interactions of hadrons and nuclei [12-18]. All these studies showed qualitative predictions of the measured parameters [19-23].

Analogous to the theoretical approach based on different views, development in the artificial intelligence (AI) field has given the neural networks a strong presence in high-energy physics [24-27]. Neural networks are composed of simple interconnected computational elements operating in parallel. These artificial neural networks (ANNs) are trained, so that a particular input leads to a specific target output. The objective of this paper is to extract the multiplicity distribution of charged pions for $h-A$ collisions at 200 GeV/c using NNM compared to PTFM. Section 2 presents parton two fireball model PTFM at high energies for the multiparticle production in hadron-nucleus $h-A$ collisions. The NN model is described in Sections 3, 4. The results and discussion of both models are compared in Section 5.

2 Parton Two Fireball Model (PTFM)

Multiplicity of the created charged particles and other parameters in h-A interactions can be determined only by the overlapping volume participating in the interaction at a given impact parameter [12, 14, 17, 18].

Let us assume that a proton with mass m and radius r_0 is incident on a nucleus of radius R .The overlapping volume at any impact parameter, V (b), is given by,

$$V(b)=\pi(r_0+R-b)\left[\frac{2}{3}r_0^{\,2}-\frac{1}{3}r_0(b-R)-\frac{1}{3}r_0(b-R)^2\right] \quad (1)$$

If we define a dimensionless impact parameter $x=\frac{b}{(r_0+R)}$,

then the fraction of partons from the projectile that participate in the interaction at a given impact parameter, Z(x), can be given by,

$$Z(x) = (\frac{1}{2} + \frac{3}{4}A^{1/3} - \frac{1}{4}A) + \frac{3}{4}(A^{2/3} + A - A^{1/3} - 1)\,x$$

$$-\frac{3}{4}(A^{1/3} + A + 2A^{2/3})x^2 + \frac{3}{4}(\frac{1}{3} + \frac{1}{3}A + A^{2/3} + A^{1/3})x^3 \quad (2)$$

and the statistical impact parameter distribution is given by

$$P(x)\,dx = 2\,x\,dx \quad (3)$$

The total probability of peripheral collisions $P_{Pr.}$ will be given by,

$$P_{Pr.}(x) = \int_a^1 2\,x\,dx \quad (4)$$

Where, $a = \frac{(A^{1/3} - 1)}{(A^{1/3} + 1)}$

The total probability of central collisions will be given by

$$P_{Cent.} = \frac{(A^{1/3} - r_0)^2}{(A^{1/3} + r_0)^2} \quad (5)$$

From equations (2) and (3) using least square fitting technique Z function distribution can be written in the form [14, 17],

$$P(z)\,dz = 2 \sum_{k=0}^{k=9} C_k Z^k \sum_{k=0}^{k=8} k C_k Z^{k-1} dZ \quad (6)$$

Where, C_k values are represented in table (1),

2.1 Charged Pion Production For Hadron- Nucleus

TABLE 1
C_k VALUES FOR CONSIDERED INTERACTIONS

Collision Type	$p^{\pm} - Ar^{40}$	$p^{\pm} - Xe^{131}$	$p^{\pm} - Au^{197}$
$C_9 =$	-318.4882903	0	-206.4492348
$C_8 =$	1433.1975229	-13.252443	929.0215018
$C_7 =$	-2718.5422389	33.4122421	-1762.202021
$C_6 =$	2826.6421281	-18.4012711	1832.2735619
$C_5 =$	-1755.406211	-21.3332567	-1137.881365
$C_4 =$	666.0383358	33.3692337	431.7361847
$C_3 =$	-152.2887535	-18.035544	-98.7158647
$C_2 =$	20.2506544	4.8202546	13.1267791
$C_1 =$	-1.8432316	-0.8927745	-1.1948108
$C_0 =$	0.9937953	0.9904029	0.995978

Collisions

The number of created pions from each fireball (n_o) will be given by,

$$n_o(Z) = \frac{Z(x)T_0}{\varepsilon} = \frac{Z(x)Q}{2\varepsilon} \quad (7)$$

Where, Q is the free energy which is carried by the two fireballs, ε is the energy required for the creation of one pion.
From Eqs. (6, 7), the probability of the emission of any number of pions (n_0) from one fireball in the peripheral collision can be obtained in the form:

$$P(n_o)dn_0 = \int_{n_0}^{n_0+1} 2 \sum_{k=0}^{k=9} C_k \left(\frac{2\varepsilon n_0}{Q}\right)^k \sum_{k=0}^{k=8} k C_k \left(\frac{2\varepsilon n_0}{Q}\right)^{k-1} \frac{2\varepsilon}{Q} dn_0 \quad (8)$$

It is clear that at a given impact parameter, Eq. (7) gives the total number of created particles (i.e. charged and neutral particles).

According to the above scheme, the charged multiplicity distribution will be given by,

$$P(n_{ch}) = \sum_{n=1}^{nch} \Phi(n)\Phi(n_{ch} - n) \quad (9)$$

$$; \quad n_{ch} = 2,4,6,........,Q/\varepsilon$$

Where, $\Phi(n) = \sum_{n_0} \Psi(n_2)\,P\,(n_o)$

, $\Psi(n_2)$ is the Poisson distribution of the form,

$$\Psi(n_2) = \frac{N^{n_2}}{n_2!} p^{n_2} e^{-NP} \quad (10)$$

Where, N: is the number of pairs of created particles from one fireball ($N = \frac{n_0}{2}$), n_2 the number of pairs of charged pions, $n_2 = \frac{n-1}{2}$, P the probability that the pair of pions is charged.
The number of negative particles from one fireball equals the half of new created charged pions $n_- = \frac{n_{ch}}{2}$

The probability distribution of negative particles $P(n_-)$ is the same as the probability distribution of charged particles

$$P(n_-) = P(n_{ch} = 2n_-) \quad (11)$$

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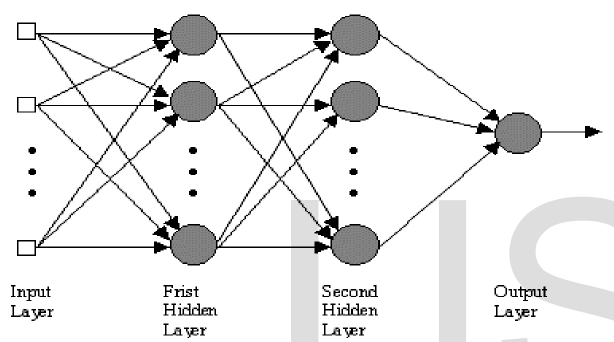
$$; \; n\_ = 0,1,2,3,........, Q/2\varepsilon$$

3 Artificial Neural Networks (ANNs)

ANNs are composed of interconnecting artificial neurons (programming constructs that mimic the properties of biological neurons). Artificial neural networks may either be used to gain an understanding of biological neural networks, or for solving artificial intelligence problems without necessarily creating a model of a real biological system. The real, biological nervous system is highly complex: artificial neural network algorithms attempt to abstract this complexity and focus on what may hypothetically matter most from an information processing point of view.

The neuron transfer function, f, is typically sigmoid or step function that produces a scalar output (n) as in Eq. (12):

$$n = f \sum_i w_i I_i + b \qquad (12)$$



"Figure 1. Neuron model"

Where, I_i, w_i, b are the i^{th} input, the i^{th} weight and b the bias respectively.

A network consists of one or more layers of neurons. A layer of neurons is a number of parallel neurons. These layers are configured in a highly interconnected topology as shown in figure (1).

4 Training of The h-a-ANN

Neural network can be trained to perform a particular function by adjusting the values of the connections (weights) between elements. Training in simple is to make a particular input leads to a specific target output. The weights are adjusted, based on a comparison of the output and the target, until the network output matches the target. Typically many such input/target pairs are used, in this supervised learning, to train a network.

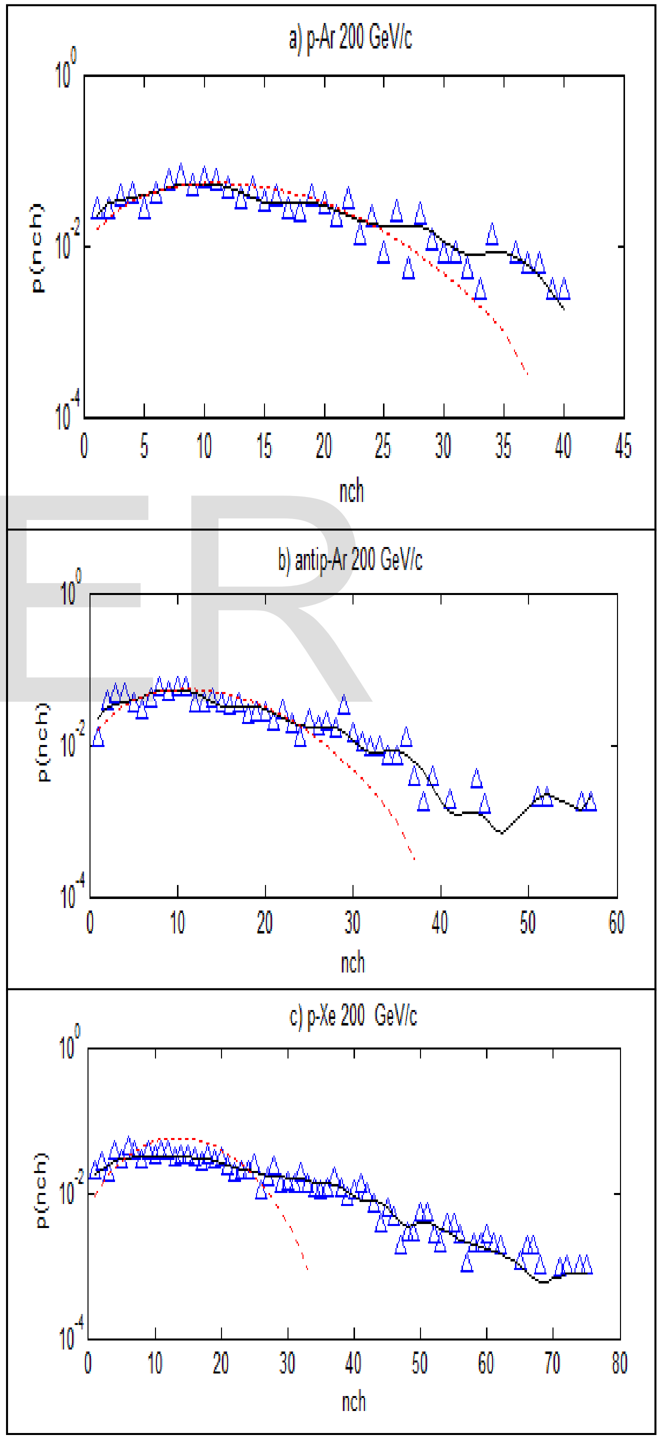
The proposed ANNs in this paper was trained using Levenberg–Marquardt optimization technique. This optimization technique is more powerful than the conventional gradient descent techniques [27-31].

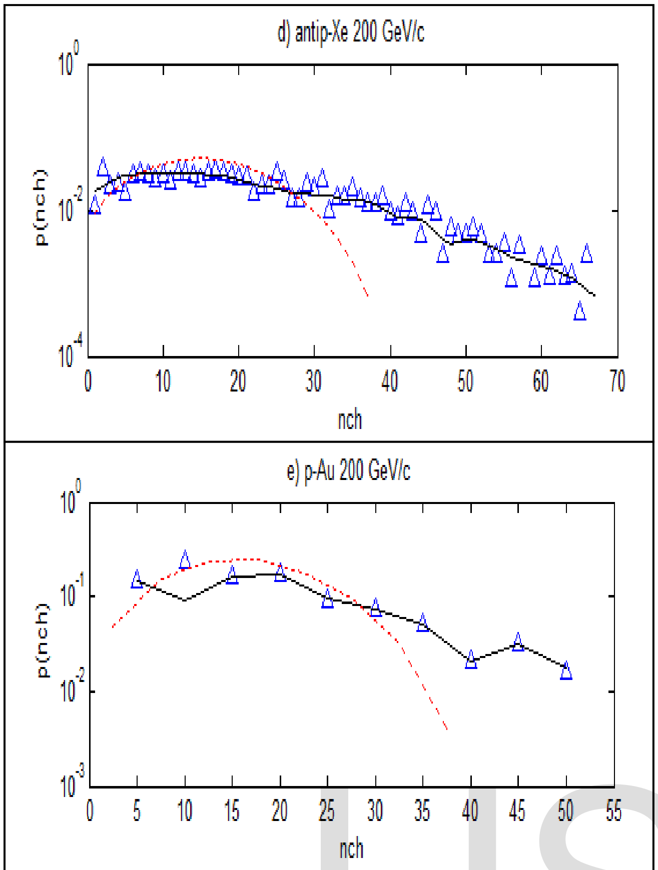
The Levenberg–Marquardt updates the network weights using the following rule,

$$\Delta W = (J^T J + \mu I)^{-1} J^T e$$

Where, J is the Jacobean matrix of derivatives of each error with respect to each weight. J^T is the transposed matrix of J; I is the identity matrix that has the same dimensions of $J^T J$, μ is a scalar; changed adaptively by the algorithm and e is an error vector.

The only requirement for this method is a considerably large memory for large problems. The initial training weights were also chosen using the Nguyen–Widrow random generator in order to speed up the training process [27-31].





"Figure 2. Comparison between the experimental and simulated multiplicity distribution of charged pions $P(n_{ch})$ for $h-A$ collisions at 200 GeV/c:(—) NN model, (……) PTFM, (△) experimental data"

5 Results And Discussion

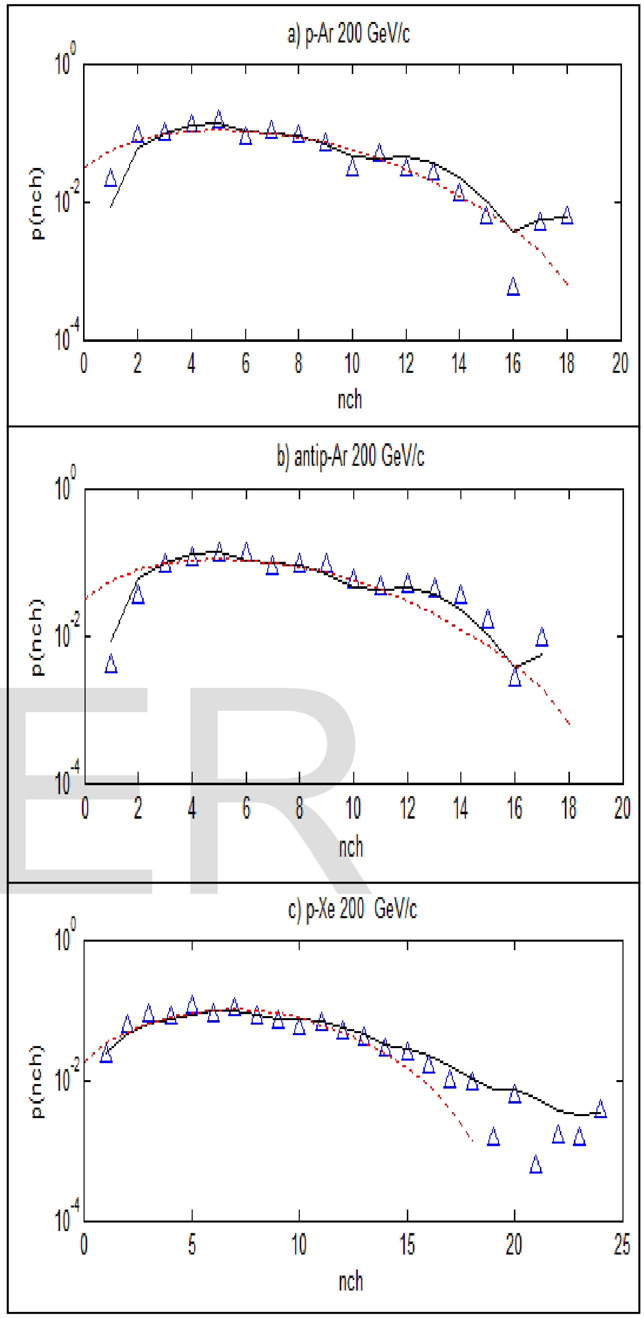
Charged and negative pions multiplicity distributions, Eq. (9, 11), are calculated for $p^{\pm}-Ar^{40}$, $p^{\pm}-Xe^{131}$, $p-Au^{197}$ and $p-He^{4}$ assuming ε in Eq. (11) is given by :

$$\varepsilon = a\,n_o + b$$

Where, a = 0.04, b= 0.35 as in references [14, 17, 18]. The results of these calculations are represented in figure 2 (a, b, c, d, e) and figure 3 (a, b, c, d, e, f) along with experimental data [32, 34] which show fair agreement with the corresponding experimental data. It can be seen from figs. (2, 3) that charged and negative pions multiplicity distributions are not in accordance with the experimental data for heavy nuclei although the situation becomes better for the light ones. The emission of secondary particles is assumed to follow a Poisson distribution. As mass number increases the multiplicity distribution is not broaden but its peak is shifted to high numbers.

We have also calculated the same collisions by using ANN model and these calculations are represented in figs (2, 3) along with the same experimental data [32-34]. We have also found great variations compared to PTFM.

Different configurations of network structure were investigated to achieve good mean squared error (MSE) and good performance for the network using the input-output arrangement. The input and target vectors are randomly divided into three sets (validation set, training set, testing set), 80% of the vectors are used to train the network and 20% of the vectors are used to validate how well the network generalized.



The proposed neural network model of charged and negative pions multiplicity distributions of $p^{\pm}-Ar^{40}$, $p^{\pm}-Xe^{131}$, $p-Au^{197}$ and $p-He^{4}$ collisions at 200 GeV/c have three inputs (n_{ch}, P_{Lab}, A), one output $P(n_{ch})$ and two hidden layers of 23, 22 neurons, for charged pions, two hidden layers of 24, 21 neurons for negative ones. The transfer functions of the first and second hidden layers were chosen to be a tan sigmoid, while the output layer was chosen to be a pure line. The trained method which used to train the ANN model is Levenberg-Marquardt optimization technique, with number of epochs=19, performance of order 10^{-5} for charged pion production and epochs=10, performance of order 10^{-4} for negative ones.

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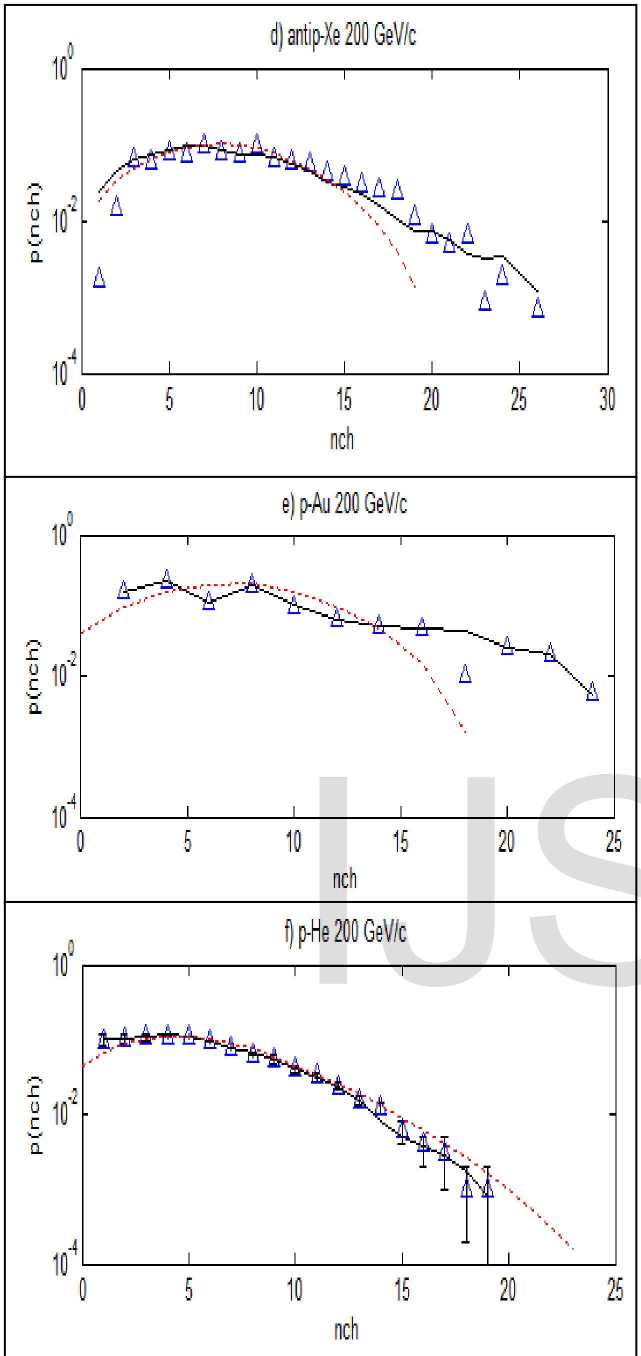

"Figure 3. Comparison between the experimental and simulated multiplicity distribution of pions $P(n\_)$ for $h-A$ collisions at 200 GeV/c: $(-)$ NN model, (……) PTFM, (△) experimental data"

It is should be emphasized that when the mass increases the multiplicity distributions of charged and negative pion production using ANN are consistent with all regions of the experimental data (low, medium, high multiplicity). In contrary with PTFM, the theoretical calculations are inconsistent with the experimental data especialy at high multiplicity. That is why; we use the ANN technique because it is able to exactly model the multiplicity distribution for different beams in hadron nucleus interactions.

APPENDIX

Our obtained function for charged and negative pions in h-A interactions is generated using the obtained control NN parameters as follows:
The structure of the network is 3-23 -22-1 for charged pions and 3-24 -21-1 for negative ones. The obtained equation which describes the multiplicity distribution of charged and negative pions in h-A interactions for different beams (projectiles), different mass numbers (nucleii) at the same energy is given by:

$$P(n_{ch}) = pureline[\{net.LW(3,2).\tan sigmoid.\{net.LW(2,1).\tan sigmoid\{net.IW(1,1).P + net.b(1)\} + net.b(2)\} + net.b(3)\}]$$

Where, pure line is linear transfer function, tan sigmoid is hyperbolic tangent sigmoid transfer function.
P is the input which is (n_{ch}, P_{Lab}, A).
$net.LW(3,2)$ linked weight between the second hidden layer and the output.
$net.LW(2,1)$ linked weights between the first and the second hidden layer.
$net.IW(1,1)$ linked weights between the input layer and the first hidden layer.
$net.b(1)$ is the bias of the first hidden layer.
$net.b(2)$ is the bias of the second hidden layer.
$net.b(3)$ is the bias of the output layer.

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